電気伝導度σの導出

フーリエ変換した電子のジャイロ運動論方程式から出発する.

$$\left[i(\omega - k_\parallel v_\parallel) + \frac{\nu_{ei}(v)}{2}\frac{\partial}{\partial\xi}(1-\xi^2)\frac{\partial}{\partial\xi}\right]h_{ek_y}$$
$$= i\omega\frac{e_e F_{Me}}{T_e}\left(1-\frac{\omega_{*e}}{\omega}-\frac{\omega_{*e}\eta_e}{\omega}\left(\frac{v^2}{2v_{the}^2}-\frac{3}{2}\right)\right)$$
$$\times\left(\phi_{k_y} - v_\parallel A_{\parallel k_y}\right) \qquad -(1)$$

~~補足~~ Collision ~~項~~モデルとして Lorentz ~~operator~~ オペレータを用いている.

また $k_\perp\rho_e \to 0$, $k_\perp\rho_i \ll 1$ (イオン、電子のFLR効果はムシ)より

$$\langle\psi_{k_y}\rangle_e \simeq \phi_{k_y} - v_\parallel A_{\parallel k_y}$$

Legendre多項式展開

h_{ek_y} を Legendre多項式で展開する.

$$h_{ek_y} = \sum_n h_n(v)P_n(\xi) \quad , \ \xi = v_\parallel/v.$$

Legendre多項式 $P_n(\xi)$ は以下をみたす.

$$\frac{\partial}{\partial\xi}(1-\xi^2)\frac{\partial}{\partial\xi}P_n(\xi) = -n(n+1)P_n(\xi) \ , \qquad P_0 = 1,\ P_1 = \xi,\ P_2 = \frac{3\xi^2-1}{2}$$

(1)は.

$$\sum_n\left[i(\omega - k_\parallel v P_1) - \frac{\nu_{ei}}{2}n(n+1)\right]h_n P_n$$
$$= i\omega\frac{e_eF_{Me}}{T_e}\left(1-\frac{\omega_{*e}}{\omega}-\cdots\right)\phi_{k_y}P_0$$
$$- i\omega\frac{e_eF_{Me}}{T_e}\left(1-\frac{\omega_{*e}}{\omega}-\cdots\right)vA_\parallel P_1$$

Legendre多項式の多項式 $xP_n(x) = \dfrac{(n+1)P_{n+1}(x)+nP_{n-1}(x)}{2n+1}$ を用いると

$$(\text{左辺}) = \sum_n h_n\left[\left(i\omega - \frac{\nu_{ei}}{2}n(n+1)\right)P_n - ik_\parallel v\frac{(n+1)P_{n+1}+nP_{n-1}}{2n+1}\right]$$

. Legendre polynomials による展開は引続き~~[illegible]~~にまとめる.

$$h_0 = \frac{k_\parallel v}{3\omega} h_1 + \left(1 - \frac{\omega_{*e}}{\omega} \cdots\right) \frac{e_e}{T_e} \phi_{k_y} F_{Me}.$$

$$\left\{\begin{aligned}
& i\omega h_0 - \frac{i k_\parallel v}{3} h_1 = i\omega\left(1 - \frac{\omega_{*e}}{\omega} - \frac{\omega_{*e}\eta_e}{\omega}\left(\frac{v^2}{2v_{the}^2} - \frac{3}{2}\right)\right)\frac{e_e}{T_e}\phi_{k_y} F_{Me} \quad \text{— (ii)} \\
& (i\omega - \nu_{ei}) h_1 - i k_\parallel v h_0 - i k_\parallel v \frac{2}{5} h_2 = -i\omega\left(1 - \frac{\omega_{*e}}{\omega} - \frac{\omega_{*e}\eta_e}{\omega}(\cdots)\right)\frac{e_e}{cT_e} v A_{\parallel k_y} F_{Me} \quad \text{— (iii)} \\
& h_{e2} = \frac{i k_\parallel v}{i\omega - 3\nu_{ei}} \frac{2}{3} h_1 \quad \text{— (iv)}
\end{aligned}\right.$$

(ii), (iv) を (iii) へ代入し整理.

$$(i\omega - \nu_{ei}) h_1 - i k_\parallel v \left(\frac{k_\parallel v}{3\omega} h_1 + (1 - \cdots)\frac{e_e}{T_e}\phi_{k_y} F_{Me}\right)$$

$$- k_\parallel v \frac{2}{5}\left(\frac{i k_\parallel v}{i\omega - 3\nu_{ei}} \frac{2}{3} h_1\right) = -i\omega\left(1 - \frac{\omega_{*e}}{\omega} \cdots\right)\frac{e_e}{cT_e} v A_{\parallel k_y} F_{Me}$$

$\left|\frac{k_\parallel v}{i\omega - \nu_{ei}}\right| \ll 1$ なら無視して良い…?

$$\left[(i\omega - \nu_{ei}) - i k_\parallel v \frac{k_\parallel v}{3\omega} \boxed{- i k_\parallel v \frac{4}{15} \frac{i k_\parallel v}{i\omega - 3\nu_{ei}}}\right] h_1$$

$$= -\left(1 - \frac{\omega_{*e}}{\omega} \cdots\right)\frac{e_e}{T_e} v F_{Me} \underbrace{\left(\frac{i\omega}{c} A_{\parallel k_y} - i k_\parallel \phi_{k_y}\right)}_{= E_{\parallel k_y}}$$

h_2 に対応する項を無視できるとして

$$h_1 = -\frac{e_e}{T_e} v F_{Me} \frac{\left(1 - \frac{\omega_{*e}}{\omega} - \frac{\omega_{*e}\eta_e}{\omega}\left(\frac{v^2}{2v_{the}^2} - \frac{3}{2}\right)\right)}{(i\omega - \nu_{ei}) - \frac{i k_\parallel^2 v^2}{3\omega}} E_{\parallel k_y}$$

$$= -i\omega \frac{e_e}{T_e} v F_{Me} \frac{\left(1 - \frac{\omega_{*e}}{\omega} - \cdots\right)}{i\omega(i\omega - \nu_{ei}) + \frac{k_\parallel^2 v^2}{3}} E_{\parallel k_y} \quad \text{— (v)}$$

アンペールの法則

イオンのパラレル流速 $J_{\parallel i}$ がかんでおとき。

$$J_{\parallel i} = e_e \int v_\parallel h_{eff} dv \qquad , \left(h_{eff} = \sum_n h_n(v) P_n(\xi) \right)$$

$$= e_e \int v_\parallel \sum_n h_n P_n dv \qquad , \left(\frac{v_\parallel}{v} = \xi \right)$$

$$= e_e \int v \xi \sum_n h_n P_n v_\perp dv_\parallel dv_\perp \cdot 2\pi .$$

$$= 2\pi e_e \int v^2 (\sqrt{1-\xi^2}) \underset{\downarrow\, P_1}{\xi} \sum_n h_n P_n \, dv_\parallel dv_\perp \qquad , \left(v_\perp = v\sqrt{1-\xi^2} \right)$$

$dv_\parallel dv_\perp \to J\, dv d\xi$ へ変換する。J はヤコビアン。

$$J = \frac{\partial (v_\perp , v_\parallel)}{\partial (v , \xi)} = \frac{\partial \left(v\sqrt{1-\xi^2} , v\xi \right)}{\partial (v , \xi)}$$

$$= \begin{vmatrix} \frac{\partial v\sqrt{1-\xi^2}}{\partial v} & \frac{\partial v\xi}{\partial v} \\ \frac{\partial}{\partial \xi} v\sqrt{1-\xi^2} & \frac{\partial}{\partial \xi} v\xi \end{vmatrix}$$

積分区間は $\int_{-\infty}^{\infty} dv_\parallel \int_0^{\infty} dv_\perp$

$\Rightarrow \int_0^{\infty} dv \int_{-1}^{1} d\xi$

何故…?

$$= v\sqrt{1-\xi^2} + \frac{v\xi^2}{\sqrt{1-\xi^2}} = \frac{v}{\sqrt{1-\xi^2}}$$

∴

$$J_{\parallel i} = 2\pi e_e \int_{-1}^{1}\int_0^{\infty} J\, dv\, d\xi \; v^2 (\sqrt{1-\xi^2}) P_1 \sum_n h_n P_n$$

$$= 2\pi e_e \int_0^{\infty} v^3 dv \int_{-1}^{1} \sum_n h_n P_1 P_n \, d\xi \qquad ,$$

$$\boxed{\text{直交性} \quad \int_{-1}^{1} P_n P_m d\xi = \frac{2}{2m+1}}$$

$$= \frac{4}{3}\pi e_e \int_0^{\infty} v^3 h_1 dv \qquad \text{— (vi)}$$

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h_1 は ~~式~~ で求めている。

P.②の(v)

(v), (vi) より σ を求める.

($j_{\parallel} = \sigma E_{\parallel}$ の関係を使って)

$$j_{\parallel k_y} = \frac{4}{3}\pi e_e \int_0^\infty v^3 \left[-i\omega \frac{e_e}{T_e} v F_{M,e} \frac{\left(1 - \frac{\omega_{*e}}{\omega} - \frac{\omega_{*e}\eta_e}{\omega}\left(\frac{v^2}{2v_{the}^2} - \frac{3}{2}\right)\right)}{i\omega(i\omega - \nu_{ei}) + \frac{k_\parallel^2 v^2}{3}} E_{\parallel k_y} \right] dv$$

$$F_{M,e} = \frac{n_e}{\pi^{3/2}(\sqrt{2}v_{the})^3} e^{-\frac{v^2}{2v_{the}^2}}$$

$$= \boxed{-i\frac{4}{3}\pi \frac{e_e^2}{T_e} \frac{n_e}{\pi^{3/2} 2\sqrt{2} v_{the}^3} \int_0^\infty v^4 e^{-\frac{v^2}{2v_{the}^2}} \frac{\omega\left(1 - \frac{\omega_{*e}}{\omega} - \cdots\right)}{i\omega(i\omega - \nu_{ei}) + \frac{k_\parallel^2 v^2}{3}} dv} \; E_{\parallel k_y}$$

σ

$$\hat{v} = \frac{v}{\sqrt{2}v_{the}}, \quad \hat{v}^4 = \frac{v^4}{4v_{the}^4}, \quad \hat{v}^2 = \frac{v^2}{2v_{the}^2}, \quad d\hat{v} \cdot \sqrt{2}v_{the} = dv$$

を導入する.

よって

$$\sigma = -\frac{\overset{2}{\cancel{4}} i}{3\sqrt{\pi}} \frac{e_e^2}{T_e} \frac{n_e}{\cancel{2\sqrt{2}} \cancel{v_{the}^3}} \int_0^\infty \hat{v}^4 \cdot 4 v_{the}^{\cancel{4}} e^{-\hat{v}^2} \frac{\omega(1 - \cdots)}{i\omega(\cdots) + \frac{k_\parallel^2 v^2}{3}} d\hat{v} \cdot \cancel{\sqrt{2}} v_{the}$$

$$= -\frac{8i}{3\sqrt{\pi}} \frac{e_e^2}{\cancel{T_e}} \frac{n_e \cancel{T_e}}{m_e} \int_0^\infty \hat{v}^4 e^{-\hat{v}^2} \frac{\omega(1 - \cdots)}{i\omega(\cdots) + \frac{k_\parallel^2 v^2}{3}} d\hat{v}$$

$$= -\frac{8}{3\sqrt{\pi}} \frac{1}{4\pi} \frac{4\pi e_e^2 n_e}{m_e} \int_0^\infty \frac{i\omega\left(1 - \frac{\omega_{*e}}{\omega} - \cdots\right)}{i\omega(i\omega - \nu_{ei}) + \frac{k_\parallel^2 v^2}{3}} \hat{v}^4 e^{-\hat{v}^2} d\hat{v}$$

$$\left(\omega_{pe}^2 = \frac{4\pi e_e^2 n_e}{m_e}\right)$$

$$= -\frac{8}{3\sqrt{\pi}} \left(\frac{\omega_{pe}^2}{4\pi}\right) \int_0^\infty \frac{i\omega}{-i\omega\nu_{ei}\left(1 - i\frac{\omega}{\nu_{ei}}\right) + \nu_{ei} k_\parallel^2 D} \left(1 - \frac{\omega_{*e}}{\omega} \cdots\right) \hat{v}^4 e^{-\hat{v}^2} d\hat{v}$$

$$D \equiv \frac{v^2}{3\nu_{ei}}$$

$$\cancel{= -\frac{8}{3\sqrt{\pi}} \left(\frac{\omega_{pe}^2}{4\pi}\right) \int_0^\infty \frac{-i\omega}{i\nu_{ei}\omega\left(1 - i\frac{\omega}{\nu_{ei}}\right) + \frac{k_\parallel^2 D}{\omega}} \left(1 - \cdots\right) \hat{v}^4 e^{-\hat{v}^2} d\hat{v}}$$

$$\boxed{\sigma = +\frac{8}{3\sqrt{\pi}} \left(\frac{\omega_{pe}^2}{4\pi}\right) \int_0^\infty \frac{i\omega/\nu_{ei}}{i\omega\left(1 - i\frac{\omega}{\nu_{ei}}\right) + k_\parallel^2 D} \left(1 - \frac{\omega_{*e}}{\omega} - \frac{\omega_{*e}\eta_e}{\omega}\left(\hat{v}^2 - \frac{3}{2}\right)\right) \hat{v}^4 e^{-\hat{v}^2} d\hat{v}}$$

※ $\nu_{ei}(v) = \frac{3\sqrt{\pi}}{4} \nu_c \left(\frac{v}{v_{the}}\right)^{-3}$ $\qquad \hat{v} = \frac{v}{\sqrt{2}v_{the}}$

$$= \frac{3\sqrt{\pi}}{4} \nu_c \left(\sqrt{2}\hat{v}\right)^{-3} = \cancel{\frac{3\sqrt{\pi}}{8\sqrt{2}} \frac{\nu_c}{\hat{v}^3}}$$

σ に含まれる速度積分の計算

- ν_{ei} にも v が含まれることに注意.

$$\nu_{ei}(v) = \frac{3}{4}\sqrt{\pi}\,\nu_c\left(\frac{v_{the}}{v}\right)^3$$

今. $\frac{v}{\sqrt{2}v_{the}} = \hat{v}$ としているので $\frac{v}{v_{the}} = \sqrt{2}\hat{v}$.

よって
$$\nu_{ei}(\hat{v}) = \frac{3}{4}\sqrt{\pi}\,\nu_c\left(\frac{1}{\sqrt{2}\hat{v}}\right)^3$$
$$= \frac{3}{4}\sqrt{\pi}\left(\frac{\nu_c}{2\sqrt{2}}\right)\left(\frac{1}{\hat{v}}\right)^3$$

表記のかんたん化のため以下を定義しておくと

$$\hat{\nu}_c \equiv \frac{\nu_c}{2\sqrt{2}}$$

このとき
$$\boxed{\nu_{ei}(\hat{v}) = \frac{3}{4}\sqrt{\pi}\,\hat{\nu}_c\,(\hat{v})^{-3}}$$

σ を再掲

$$\sigma = \frac{8}{3\sqrt{\pi}}\left(\frac{\omega_{pe}^2}{4\pi}\right)\int_0^\infty \frac{i\omega}{\nu_{ei}}\frac{1}{i\omega\left(1-i\frac{\omega}{\nu_{ei}}\right)+k_{\parallel}^2 D}\left(1-\frac{\omega_{*e}}{\omega}-\frac{\omega_{*e}}{\omega}\eta_e\left(\hat{v}^2-\frac{3}{2}\right)\right)$$
$$\times\,\hat{v}^4 e^{-\hat{v}^2}d\hat{v}$$

衝突が十分に強い領域を考えるとき. $k_{\parallel}^2 D$ 項は無視できる.

$\frac{\nu_{ei}}{\omega} \gg 1$ に対して

$$\sigma \simeq \frac{8}{3\sqrt{\pi}}\left(\frac{\omega_{pe}^2}{4\pi}\right)\int_0^\infty \frac{\cancel{i\omega}}{\nu_{ei}}\frac{1}{\cancel{i\omega}\left(1-i\frac{\omega}{\nu_{ei}}\right)}\left(1-\frac{\omega_{*e}}{\omega}-\cdots\right)e^{-\hat{v}^2}\hat{v}^4 d\hat{v}$$

$$\simeq \frac{8}{3\sqrt{\pi}}\left(\frac{\omega_{pe}^2}{4\pi}\right)\boxed{\int_0^\infty \frac{1}{\nu_{ei}}\left(1+i\frac{\omega}{\nu_{ei}}\right)\left(1-\frac{\omega_{*e}}{\omega}-\cdots\right)e^{-\hat{v}^2}\hat{v}^4 d\hat{v}}$$

↑

速度積分を I とおく.

$$I \equiv \int_0^\infty \frac{1}{\nu_{ei}(\hat{v})}\left(1+i\frac{\omega}{\nu_{ei}(\hat{v})}\right)\left(1-\frac{\omega_{*e}}{\omega}-\frac{\omega_{*e}}{\omega}\eta_e\left(\hat{v}^2-\frac{3}{2}\right)\right)e^{-\hat{v}^2}\hat{v}^4 d\hat{v}$$

$$I = \underbrace{\int_0^\infty \frac{4}{3\sqrt{\pi}} \frac{1}{\hat{\nu}_c}\left(1-\frac{\omega_{*e}}{\omega}\left(1-\frac{3}{2}\eta_e\right)-\frac{\omega_{*e}}{\omega}\eta_e \hat{u}^2\right)e^{-\hat{u}^2}\hat{u}^7 d\hat{u}}_{I_1}$$
$$+ i\underbrace{\int_0^\infty \left(\frac{4}{3\sqrt{\pi}}\right)^2 \frac{\omega}{\hat{\nu}_c^2}\left(1-\frac{\omega_{*e}}{\omega}\left(1-\frac{3}{2}\eta_e\right)-\frac{\omega_{*e}}{\omega}\eta_e \hat{u}^2\right)e^{-\hat{u}^2}\hat{u}^{10} d\hat{u}}_{I_2}$$

※ $\nu_{ei}(\hat{u}) = \frac{3}{4}\sqrt{\pi}\,\hat{\nu}_c(\hat{u})^{-3}$ を代入した

$\hat{\nu}_c = [redacted]\ \frac{\nu_c}{2\sqrt{2}}$

$$I_1 = \frac{4}{3\sqrt{\pi}}\frac{1}{\hat{\nu}_c}\int_0^\infty \left(1-\frac{\omega_{*e}}{\omega}\left(1-\frac{3}{2}\eta_e\right)-\frac{\omega_{*e}}{\omega}\eta_e \hat{u}^2\right)e^{-\hat{u}^2}\hat{u}^7 d\hat{u}$$
$$= \frac{4}{3\sqrt{\pi}}\frac{1}{\hat{\nu}_c}\left[\left(1-\frac{\omega_{*e}}{\omega}\left(1-\frac{3}{2}\eta_e\right)\right)\underbrace{\int_0^\infty e^{-\hat{u}^2}\hat{u}^7 d\hat{u}}_{=3} - \frac{\omega_{*e}}{\omega}\eta_e \overbrace{\boxed{\int e^{-\hat{u}^2}\hat{u}^9 d\hat{u}}}^{=12}\right]$$
$$= \frac{4}{3\sqrt{\pi}}\frac{3}{\hat{\nu}_c}\left(1-\frac{\omega_{*e}}{\omega}\left(1-\frac{3}{2}\eta_e\right)-4\frac{\omega_{*e}}{\omega}\eta_e\right)$$
$$= \frac{4}{3\sqrt{\pi}}\frac{3}{\hat{\nu}_c}\left(1-\frac{\omega_{*e}}{\omega}\left(1+\frac{5}{2}\eta_e\right)\right)$$

$-\frac{8}{2}+\frac{3}{2} = -\frac{5}{2}$

$$I_2 = \left(\frac{4}{3\sqrt{\pi}}\right)^2\frac{\omega}{\hat{\nu}_c^2}\int_0^\infty \left(1-\frac{\omega_{*e}}{\omega}\left(1-\frac{3}{2}\eta_e\right)-\frac{\omega_{*e}}{\omega}\eta_e \hat{u}^2\right)e^{-\hat{u}^2}\hat{u}^{10} d\hat{u}$$
$$= \left(\frac{4}{3\sqrt{\pi}}\right)^2\frac{\omega}{\hat{\nu}_c^2}\left[\left(1-\frac{\omega_{*e}}{\omega}\left(1-\frac{3}{2}\eta_e\right)\right)\underbrace{\int_0^\infty e^{-\hat{u}^2}\hat{u}^{10} d\hat{u}}_{=\frac{945}{64}\sqrt{\pi}} - \frac{\omega_{*e}}{\omega}\eta_e \underbrace{\boxed{\int_0^\infty e^{-\hat{u}^2}\hat{u}^{12} d\hat{u}}}_{=\frac{10395}{128}\sqrt{\pi}}\right]$$
$$= \left(\frac{4}{3\sqrt{\pi}}\right)^2\frac{\omega}{\hat{\nu}_c^2}\frac{945\sqrt{\pi}}{64}\left[1-\frac{\omega_{*e}}{\omega}\left(1-\frac{3}{2}\eta_e\right)-\frac{11}{2}\frac{\omega_{*e}}{\omega}\eta_e\right]$$
$$= \left(\frac{4}{3\sqrt{\pi}}\right)^2\frac{\omega}{\hat{\nu}_c^2}\frac{945\sqrt{\pi}}{64}\left(1-\frac{\omega_{*e}}{\omega}\left(1+4\eta_e\right)\right)$$

$\frac{3}{2}-\frac{11}{2}$

$$= \left(\frac{4}{3\sqrt{\pi}}\frac{3}{\hat{\nu}_c}\right)\cdot\frac{4}{3\sqrt{\pi}}\frac{\omega}{\hat{\nu}_c}\frac{315\sqrt{\pi}}{64}\left(\quad \right)$$

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$I = I_1 + i I_2$ より

$$I = \frac{4\times\cancel{3}}{\cancel{3}\sqrt{\pi}}\frac{1}{\hat{\ell}_c}\left[1-\frac{\omega_{*e}}{\omega}\left(1+\frac{5}{2}\eta_e\right)+i\,\frac{\cancel{4}}{\cancel{3\sqrt{\pi}}}\,\frac{\omega}{\hat{\ell}_c}\cdot\frac{\overset{105}{\cancel{315\sqrt{\pi}}}}{\underset{16}{\cancel{64}}}\left(1-\frac{\omega_{*e}}{\omega}(1+4\eta_e)\right)\right]$$

電気伝導度 σ

$$\sigma = \frac{8}{3\sqrt{\pi}}\left(\frac{\omega_{pe}^2}{4\pi}\right)\times I$$

$$= \frac{8}{3\sqrt{\pi}}\left(\frac{\omega_{pe}^2}{4\pi}\right)\frac{4}{\sqrt{\pi}}\frac{1}{\hat{\ell}_c}\underbrace{\left[1-\frac{\omega_{*e}}{\omega}\left(1+\frac{5}{2}\eta_e\right)+\frac{i\omega}{\hat{\ell}_c}\frac{105}{16}\left(1-\frac{\omega_{*e}}{\omega}(1+4\eta_e)\right)\right]}_{X}$$

$$= \underbrace{\frac{32}{3\pi}\frac{1}{\hat{\ell}_c}\left(\frac{\omega_{pe}^2}{4\pi}\right)}_{=\sigma_L} X$$

$$\boxed{\sigma_L \equiv \frac{32}{3\pi}\frac{1}{\hat{\ell}_c}\left(\frac{\omega_{pe}^2}{4\pi}\right)} = \cancel{\frac{32}{3\pi}\frac{2\sqrt{2}}{\ell_c}\left(\frac{\omega_{pe}^2}{4\pi}\right)}$$

ただし, $\hat{\ell}_c = \frac{\ell_c}{2\sqrt{2}}$